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MODES OF CONTROL

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SUMMARY

A linear autonomous system with a single control variable is considered. There are, in general, several modes of control for such a system. The concepts of single component regulation and multiple component regulation are defined in the introduction. The relation between these modes of control is developed and an example is given to illustrate this relation.

INTRODUCTION

An nth order system will be considered where the equations of motion are written in the form

$$\dot{x} = Ax + bu \tag{1}$$

where

x is a column vector with elements $x_1(t)$, $x_2(t)$, . . . $x_n(t)$, which describe the state of the system,

u(t) is a scalar (the control variable),

A is a constant nxn matrix, and

b is a constant column vector

Multiple component regulation is defined as control of less than n of the state variables by bringing them to zero in a finite time and holding them zero thereafter. Single component regulation is defined as control of a single state variable by bringing this variable to zero in a finite time and holding it zero thereafter. It is seen from the definitions that single component regulation is a special case of multiple component regulation. Time-optimal multiple component regulation is defined in the obvious way when u(t) is bounded; i. e., bring the state variables to be controlled to zero in the minimum time such that they may be held at zero thereafter with u(t) satisfying the bounding constraint. Time-optimal single component regulation is discussed in ref. 1 and in section 4.

It is the purpose of this discussion to show that multiple component regulation can be accomplished by single component regulation.

DEVELOPMENT

Suppose that for a system described by (1) it is desired to control the variables $x_1, x_2, \ldots, x_m$, $1 \leq m \leq n$. If $m = 1$, the problem is the single component problem of controlling x_1. Thus, assume $m > 1$ and for convenience introduce the following notation:

$$\xi_1 = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_m \end{bmatrix} \qquad \xi_2 = \begin{bmatrix} x_{m+1} \\ x_{m+2} \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \qquad \beta_1 = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_m \end{bmatrix} \qquad \beta_2 = \begin{bmatrix} b_{m+1} \\ b_{m+2} \\ \cdot \\ \cdot \\ \cdot \\ b_n \end{bmatrix}$$

$$A_1 = \begin{bmatrix} a_{11} & \cdots & a_{1m} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{m1} & \cdots & a_{mm} \end{bmatrix} \qquad A_2 = \begin{bmatrix} a_{1,m+1} & \cdots & a_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{m,m+1} & \cdots & a_{mn} \end{bmatrix}$$

$$A_3 = \begin{bmatrix} a_{m+1,1} & \cdots & a_{m+1,m} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{n1} & \cdots & a_{nm} \end{bmatrix} \qquad A_4 = \begin{bmatrix} a_{m+1,m+1} & \cdots & a_{m+1,n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{n,m+1} & \cdots & a_{nn} \end{bmatrix}$$

Then (1) becomes

$$\dot{\xi}_1 = A_1\xi_1 + A_2\xi_2 + \beta_1 u \tag{2}$$

$$\dot{\xi}_2 = A_3\xi_1 + A_4\xi_2 + \beta_2 u$$

and it is desired that ξ_1 be controlled.

Holding ξ_1 zero requires that $\xi_1 = \dot{\xi}_1 = 0$, in which case (2) becomes

$$0 = A_2\xi_2 + \beta_1 u$$

$$\dot{\xi}_2 = A_4\xi_2 + \beta_2 u \tag{3}$$

If $\beta_1 = 0$ and $A_2 = 0$, it is impossible to control ξ_1 because the first equation of (2) becomes $\dot{\xi}_1 = A_1\xi_1$ and the proposed problem is of no interest. If $\beta_1 = 0$ and $A_2 \neq 0$, it may be assumed without loss of generality that the first row of A_2 contains a non-zero element. Then the state variable

$$y_{m+1} = \sum_{j=m+1}^{n} a_{1j}x_j$$

is necessarily controlled when ξ_1 is controlled. Thus the original problem can be reformulated so that it becomes a problem of controlling m + 1 state variables, i.e. $x_1, x_2, \ldots, x_m$ and y_{m+1}.

Hence, consider the case when $\beta_1 \neq 0$ and assume without loss of generality that $b_m \neq 0$. Making a transformation

$$y = Sx, \text{ where } S = \begin{bmatrix} S_1 & 0 \\ 0 & I_{n-m} \end{bmatrix}$$

$$S_1 = \begin{bmatrix} b_m & 0 & \cdots & 0 & -b_1 \\ & b_m & \cdots & 0 & -b_2 \\ & & \ddots & \vdots & \vdots \\ & \bigcirc & & b_m & -b_{m-1} \\ & & & & 1 \end{bmatrix}$$

and setting

$$\zeta_1 = \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} = S_1 \xi_1 \quad \text{and} \quad \zeta_2 = \begin{bmatrix} y_{m+1} \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} x_{m+1} \\ \vdots \\ x_n \end{bmatrix} \text{ (2) becomes}$$

$$\dot{\zeta}_1 = S_1 A_1 S_1^{-1} \zeta_1 + S_1 A_2 \zeta_2 + S_1 \beta_1 u$$

$$\dot{\zeta}_2 = A_3 S_1^{-1} \zeta_1 + A_4 \zeta_2 + \beta_2 u \tag{4}$$

S_1 was chosen so that

$$S_1 \beta_1 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ b_m \end{bmatrix}$$

If the kth row, $k < m$, of $S_1 A_2$ has non-zero elements, the control of ζ_1 implies the control of the kth component of $S_1 A_2 \zeta_2$ and the original problem[1] may be reformulated so that it becomes a problem of controlling $m + 1$ state variables.

From the foregoing considerations, it is clear that if it is possible to control ξ_1 only; then (4) may be obtained with

$$
S_1\beta_1 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_m \end{bmatrix} \qquad S_1A_2 = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & & 0 \\ a^{(1)}_{m,m+1} & \cdots & a^{(1)}_{mn} \end{bmatrix} \qquad S_1A_1S_1^{-1} = \begin{bmatrix} a^{(1)}_{11} & \cdots & a^{(1)}_{1m} \\ \vdots & & \vdots \\ a^{(1)}_{m1} & \cdots & a^{(1)}_{mm} \end{bmatrix}
$$

Now if each $a^{(1)}_{im} = 0$ with $i < m$, the components $y_1, y_2, \cdots y_{m-1}$ of ζ_1 are not controllable; hence ζ_1, and consequently ξ_1, are not controllable. Thus it may be assumed without loss of generality that $a^{(1)}_{m-1,m} \neq 0$. Define

$$
S_2 = \begin{bmatrix} a^{(1)}_{m-1,m} & 0 & \cdots & 0 & -a^{(1)}_{1m} & 0 \\ & a^{(1)}_{m-1,m} & \cdot & 0 & -a^{(1)}_{2m} & 0 \\ & & \ddots & & \vdots & \vdots \\ & \bigcirc & & a^{(1)}_{m-1,m} & -a^{(1)}_{m-2,m} & 0 \\ & & & & 1 & 0 \\ & & & & & 1 \end{bmatrix}
$$

and denote the ijth element of $S_2S_1A_1S_1^{-1}S_2^{-1}$ by $a^{(2)}_{ij}$. Then $a^{(2)}_{im} = 0$, $i<m-1$ and $a^{(2)}_{m-1,m} = a^{(1)}_{m-1,m} \neq 0$. Now if each $a^{(2)}_{i,m-1} = 0$ with $i<m-1$, the first $m-2$ components of $S_2\zeta_1$ are not controllable and hence ξ_1 is not controllable. Thus it may be assumed without loss of generality that $a^{(2)}_{m-2,m-1} = 0$. Define

$$S_3 = \begin{bmatrix} a^{(2)}_{m-2,m-1} & 0 & \cdots & 0 & a^{(2)}_{1,m-1} & 0 & 0 \\ & a^{(2)}_{m-2,m-1} & & 0 & -a^{(2)}_{2,m-1} & 0 & 0 \\ & & \ddots & & \vdots & \vdots & \vdots \\ & & & a^{(2)}_{m-2,m-1} & -a^{(2)}_{m-3,m-1} & 0 & 0 \\ & & & & 1 & 0 & 0 \\ & 0 & & & & 1 & 0 \\ & & & & & & 1 \end{bmatrix}$$

Continuing this process, $S^* = S_{m-1}\ S_{m-2} \cdots S_2S_1$ is determined so that

$$S^*A_1(S^*)^{-1} \text{ has the form } \begin{bmatrix} \alpha_{11} & \alpha_{12} & 0 & -- & -- & 0 \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & -- & -- & -- \\ -- & -- & -- & -- & -- & -- \\ -- & -- & -- & -- & -- & 0 \\ \alpha_{m-1,1} & & & & & \alpha_{m-1,m} \\ \alpha_{m1} & & & & & \alpha_{m,m} \end{bmatrix}$$

with $\alpha_{i,i+1} \neq 0$; $i = 1, 2, \ldots, m-1$. $S^*A_2 = S_1A_2$ and $S^*\beta_1 = S_1\beta_1$.

Letting $z = \begin{bmatrix} S^* & 0 \\ 0 & I_{n-m} \end{bmatrix} x$ and $\zeta_3 = \begin{bmatrix} z_1 \\ \vdots \\ z_m \end{bmatrix}$ $\quad \zeta_4 = \begin{bmatrix} z_{m+1} \\ \vdots \\ z_n \end{bmatrix}$

(2) becomes

$$\dot{\zeta}_3 = S^*A_1(S^*)^{-1}\zeta_3 + S^*A_2\zeta_4 + S^*\beta_1 u$$

$$\dot{\zeta}_4 = A_3(S^*)^{-1}\zeta_3 + A_4\zeta_4 + \beta_2 u \tag{5}$$

from (5),

$$z_{k+1} = \frac{1}{\alpha_{k,k+1}}\left[\dot{z}_k - \sum_{j=1}^{k} \alpha_{kj} z_j\right], \; k = 1, 2, \ldots, m-1.$$

With the aid of this recursive relation, it is possible to obtain z_k as a linear combination of z_1 and its first k-1 derivatives for $k \leq m$. Thus, controlling z_1 accomplishes control of ξ_1.

REMARKS

If $S_1 A_1 = 0$ (which implies $A_2 = 0$), the equations governing ξ_1 are independent of ξ_2, and z_1 satisfies an equation of the form

$$z_1^{(m)} + a_1 z_1^{(m-1)} + \ldots + a_m z_1 = bu \tag{6}$$

If ξ_1 is to be held zero after it is brought to zero,

$$u(t) = \frac{-1}{b_m} \sum_{j=m+1}^{n} a_{mj}^{(1)} x_j(t)$$

must hold as an identity in t. This is the same requirement that would have to be satisfied in order to hold z_1 to zero after $z_1, \dot{z}_1, \ldots z_1^{(m-1)}$ are brought to zero.

If the control u(t) is to satisfy a constraint such as $|u(t)| \leq 1$ for all t, then it is necessary to consider

$$\frac{1}{b_m} \sum_{j=m+1}^{n} a_{mj}^{(1)} x_j(t) \text{ for } t > 0 \text{ where } x_j(t),\ j = m+1,\ m+2, \ldots, n \text{ are}$$

determined from

$$\dot{\zeta}_2 = A_4 \zeta_2 - \beta_2 \left(\frac{1}{b_m} \sum_{j=m+1}^{n} a_{mj}^{(1)} x_j \right),\ x_j(0) = x_j^\circ,\ j-m = 1, 2, \ldots, n-m$$

From this consideration it may be that if

$$u(t) = \frac{-1}{b_m} \sum_{j=m+1}^{n} a_{mj}^{(1)} x_j(t)$$

is to satisfy $|u(t)| \leq 1$ for $t > 0$, then the x_j°'s must satisfy

$$\sum_{j=m+1}^{n} p_{rj} x_j^\circ = 0 \qquad \sum_{j=m+1}^{n} (p_{rj})^2 \neq 0 \text{ for } r = 1, 2, \ldots, q \leq n-m$$

If this occurs, the control of ξ_1 will imply that

$$y_r = \sum_{j=m+1}^{n} p_{rj} x_j$$

is also to be controlled for $r = 1, 2, \ldots, q$.

The relations that must exist between the components of ξ_1 to make it possible to control only ξ_1 can be determined by the following considerations. From (1) it is possible to obtain

$$\sum_{j=1}^{n} c_{ij} x_i^{(j)} = \sum_{j=0}^{n} d_{ij} u^{(j)} \qquad i = 1, 2, \ldots m$$

which the components of ξ_1 must satisfy. If ξ_1 is to remain zero after reaching zero at time T, then each $x_i(t)$, $i = 1, 2, \ldots m$ must be identically zero for $t > T$. This implies that $x_i^{(j)}(T) = 0, i = 1, 2, \ldots, m, \; j = 0, 1, \ldots n - 1$, and

$$\sum_{j=0}^{n} d_{ij} u^{(j)}(t) = 0 \text{ for } t > T, \; i = 1, 2, \ldots, m.$$

Thus u(t) must be a common solution of the m equations

$$\sum_{j=0}^{n} d_{ij} u^{(j)}(t) = 0, \; i = 1, 2, \ldots, m \text{ for } t > T.$$

From further considerations it can be shown that control of ξ_1 implies that p-q state variables are controlled where $p \leq n$ is such that $c_{in} \neq 0$ for some $i = 1, 2, \ldots, m$ and $c_{i,p+j} = 0$ for all $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots,$ n-p; and q is defined as the degree of the greatest common divisor of the polynomials

$$D_i = \sum_{j=0}^{n} d_{ij} s^j, \quad i = 1, 2, \ldots, m.$$

In the general situation, p = n and q = 0 so that control of ξ_1 implies control of ξ_2 as well. The notable exception to this is when ξ_1 is such that $x_i = \theta^{(i-1)}$, for $i = 1, 2, \ldots, m$. Then $D_1 = D$ and q is equal to the degree of D_1, which may be greater than zero. If $q = p - m$, then control of ξ_1 only is possible.

EXAMPLE

For clarification, the method described in the development given above was applied to the following realistic system:

$$
\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{\alpha} \\ \dot{u} \\ \dot{\delta}_e \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & -0.5997 & -0.2515 & 0.0000175 & -0.96525 \\ 0 & 1 & -0.526 & -0.001585 & -0.0803 \\ -32.2 & 0 & 13.58 & -0.0351 & 0 \\ 0 & 0 & 0 & 0 & -0.02 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ \alpha \\ u \\ \delta_e \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.2 \end{bmatrix} f, |f| \leq 1
$$

The results are listed below and then two cases are considered in detail.

It it is desired to control the following variables, then	it is necessary to control these variables:
θ	θ, $\dot{\theta}$, $-0.2515\alpha + 0.0000175u - 0.96525\,\delta_e$
$\theta, \dot{\theta}$	θ, $\dot{\theta}$, $-0.2515\alpha + 0.0000175u - 0.96525\,\delta_e$
θ, α	$\theta, \dot{\theta}, \alpha, u, \delta_e$
θ, u	$\theta, \dot{\theta}, \alpha, u, \delta_e$
θ, δ_e	$\theta, \dot{\theta}, \alpha, u, \delta_e$
$\dot{\theta}$	$\dot{\theta}$, $-0.2515\alpha + 0.0000175u - 0.96525\,\delta_e$
$\dot{\theta}, \alpha$	$\theta, \dot{\theta}, \alpha, u, \delta_e$
$\dot{\theta}, u$	$\theta, \dot{\theta}, \alpha, u, \delta_e$
$\dot{\theta}, \delta_e$	$\theta, \dot{\theta}, \alpha, u, \delta_e$
α	α, $\dot{\theta}$, $-0.001585u - 0.0803\,\delta_e$
α, u	$\theta, \dot{\theta}, \alpha, u, \delta_e$
α, δ_e	$\theta, \dot{\theta}, \alpha, u, \delta_e$
u	u, $32.20 - 13.58\alpha$, $0.526\alpha + 1.371\dot{\theta} + 0.0803\delta_e$
u, δ_e	$\theta, \dot{\theta}, \alpha, u, \delta_e$
δ_e	δ_e

The last entry is of academic interest only. If it is desired to control any three or four of the variables θ, $\dot{\theta}$, α, u, δ_e; then it is necessary to control all five of them.

Control of θ and $\dot{\theta}$ will be considered even though this is clearly single-variable control

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -0.5997 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ -0.2515 & 0.0000175 & -0.96525 \end{bmatrix} \begin{bmatrix} \alpha \\ u \\ \delta_e \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} f$$

$$\begin{bmatrix} \dot{\alpha} \\ \dot{u} \\ \dot{\delta}_e \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -32.2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} -0.528 & -0.001585 & -0.0803 \\ 13.58 & -0.0351 & 0 \\ 0 & 0 & -0.02 \end{bmatrix} \begin{bmatrix} \alpha \\ u \\ \delta_e \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.2 \end{bmatrix} f$$

Setting θ, $\dot{\theta}$ and $\dot{\theta}$, $\ddot{\theta}$ zero gives $-0.2515\alpha + 0.0000175u - 0.96525\delta_e = 0$. Therefore, let $x_1 = \theta$, $x_2 = \dot{\theta}$, $x_3 = 0.2515\alpha + 0.0000175u - 0.96525\delta_e$, $x_4 = u$, $x_5 = \delta_e$ and it is required to control x_1, x_2, and x_3.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.5997 & 1 \\ -0.000564 & -0.2515 & -0.527 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.000407 & -0.468 \end{bmatrix} \begin{bmatrix} x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -0.193 \end{bmatrix} f$$

$$\begin{bmatrix} \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} -32.2 & 0 & -54.0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -0.0342 & -52.1 \\ 0 & -0.02 \end{bmatrix} \begin{bmatrix} x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} f$$

This is the form of the equations which was sought. It is evident that it is possible to control x_1, x_2, and x_3 only and that this is accomplished by control of $\theta = x_1$. (Note that $x_3 = \ddot{\theta} + 0.5997\dot{\theta}$.) Furthermore, it is clear that when $x_1 = x_2 = x_3 = 0$ is to hold for $t > T$, $0.193f(t) = 0.00407x_4(t) - 0.468x_5(t)$ and $x_4(t)$, $x_5(t)$ satisfy

$$\dot{x}_4 = -0.0342x_4 - 52.1x_5$$

$$\dot{x}_5 = 0.00422x_4 - 0.505x_5$$

with initial conditions $x_4(T)$ and $x_5(T)$. Since the real parts of the characteristic roots for this system in x_4 and x_5 are negative, the constraint that $f \leq 1$ does not impose that a linear combination of $x_4(T)$ and $x_5(T)$ be zero.

Now the case will be considered when it is desired to control $\dot{\theta}$ and α. Let $x_1 = \dot{\theta}$, $x_2 = \alpha$, $x_3 = u$, $x_4 = \delta_e$, $x_5 = \theta$. Then

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -0.5997 & -0.2515 \\ 1 & -0.526 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0.0000175 & -0.96525 & 0 \\ -0.001585 & -0.0803 & 0 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} f$$

$$\begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} 0 & 13.58 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -0.0351 & 0 & -32.2 \\ 0 & -0.02 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.2 \\ 0 \end{bmatrix} f$$

Setting $x_1 = x_2 = \dot{x}_1 = \dot{x}_2 = 0$ gives

$$0.0000175x_3 - 0.96525x_4 = 0$$

$$0.001585x_3 - 0.0803x_4 = 0$$

At this point, $y_3 = 0.0000175x_3 - 0.96525x_4$, $y_1 = x_1$, $y_2 = x_2$, $y_4 = x_4$, $y_5 = x_5$ can be introduced. Note that control of x_1 and x_2 implies control of y_1, y_2, and y_3. However, since there are two equations, control of x_1 and x_2 implies also the control of y_3 and $(0.001585x_3 + 0.0803x_4)$; but control of the latter two variables implies control of x_3 and x_4. Thus, consider

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -0.5997 & -0.2515 & 0.0000175 & -0.96525 \\ 1 & -0.526 & -0.001585 & -0.0803 \\ 0 & 13.58 & -0.351 & 0 \\ 0 & 0 & 0 & -0.02 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -32.2 \\ 0 \end{bmatrix} x_5 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.2 \end{bmatrix} f$$

$$\begin{bmatrix} \dot{x}_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} x_5 + \begin{bmatrix} 0 \end{bmatrix} f$$

Setting $x_1 = x_2 = x_3 = x_4 = \dot{x}_1 = \dot{x}_2 = \dot{x}_3 = \dot{x}_4 = 0$ gives $-32.2x_5 = 0$. Thus, control of x_1, x_2, x_3, and x_4 implies control of all the variables x_1, x_2, x_3, x_4, and x_5.

CONCLUSIONS

It is shown by a constructive procedure that a multiple component regulation problem may be reformulated so that it becomes a single component regulation problem. This result holds for systems with a single control variable.

Systems with more than one control function are not considered. A similar analysis for such systems would seem to be worthwhile.

REFERENCE

1. Schmidt, S. F.: The Analysis and Design of Continuous and Sampled-Data Feedback Control Systems with a Saturation Type Nonlinearity. NASA TN D-20.

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